Corrigé de l'exercice 1

Donner la forme canonique des polynômes P , Q , R et S .

$P(x) = x^2 - 6x + 8$

$\quad = (x-3)^2 - 3^2 + 8$

$\quad = (x-3)^2 - 9 + 8$

$\boxed{P(x) = (x-3)^2 - 1}$

$R(x) = 25x^2 - 20x + 4$

$\quad = (5x - 2)^2$

$\quad = \left(5 \times \left(x - \dfrac{2}{5}\right)\right)^2$

$\boxed{R(x) = 25 \times \left(x - \dfrac{2}{5}\right)^2}$

$Q(x) = x^2 - 7x - 3$

$\quad = \left(x - \dfrac{7}{2}\right)^2 - \left(\dfrac{7}{2}\right)^2 - 3$

$\quad = \left(x - \dfrac{7}{2}\right)^2 + \dfrac{-49}{4} - \dfrac{3 \times 4}{1 \times 4}$

$\quad = \left(x - \dfrac{7}{2}\right)^2 + \dfrac{-49}{4} - \dfrac{12}{4}$

$\boxed{Q(x) = \left(x - \dfrac{7}{2}\right)^2 + \dfrac{-61}{4}}$

$S(x) = -4x^2 - 6x + 3$

$\quad = -4 \times \left(x^2 + \dfrac{3}{2}x - \dfrac{3}{4}\right)$

$\quad = -4 \times \left(\left(x + \dfrac{3}{4}\right)^2 - \left(\dfrac{3}{4}\right)^2 + \dfrac{-3}{4}\right)$

$\quad = -4 \times \left(\left(x + \dfrac{3}{4}\right)^2 + \dfrac{-9}{16} + \dfrac{-3 \times 4}{4 \times 4}\right)$

$\quad = -4 \times \left(\left(x + \dfrac{3}{4}\right)^2 + \dfrac{-9}{16} + \dfrac{-12}{16}\right)$

$\quad = -4 \times \left(\left(x + \dfrac{3}{4}\right)^2 + \dfrac{-21}{16}\right)$

$\quad = -4 \times \left(x + \dfrac{3}{4}\right)^2 + \dfrac{-21 \times \cancel{4} \times (-1)}{\cancel{4} \times 4}$

$\boxed{S(x) = -4 \times \left(x + \dfrac{3}{4}\right)^2 + \dfrac{21}{4}}$

Corrigé de l'exercice 2

Donner la forme canonique des polynômes P , Q , R et S .

$P(x) = 4x^2 - 6x - 7$

$\quad = 4 \times \left(x^2 - \dfrac{3}{2}x - \dfrac{7}{4}\right)$

$\quad = 4 \times \left(\left(x - \dfrac{3}{4}\right)^2 - \left(\dfrac{3}{4}\right)^2 + \dfrac{-7}{4}\right)$

$\quad = 4 \times \left(\left(x - \dfrac{3}{4}\right)^2 + \dfrac{-9}{16} + \dfrac{-7 \times 4}{4 \times 4}\right)$

$\quad = 4 \times \left(\left(x - \dfrac{3}{4}\right)^2 + \dfrac{-9}{16} + \dfrac{-28}{16}\right)$

$\quad = 4 \times \left(\left(x - \dfrac{3}{4}\right)^2 + \dfrac{-37}{16}\right)$

$\quad = 4 \times \left(x - \dfrac{3}{4}\right)^2 + \dfrac{-37 \times \cancel{4}}{\cancel{4} \times 4}$

$\boxed{P(x) = 4 \times \left(x - \dfrac{3}{4}\right)^2 + \dfrac{-37}{4}}$

$Q(x) = 9x^2 + 18x + 9$

$\quad = (3x + 3)^2$

$\quad = \left(3 \times \left(x + \dfrac{3}{3}\right)\right)^2$

$\boxed{Q(x) = 9 \times (x + 1)^2}$

$S(x) = x^2 - 14x + 9$

$\quad = (x - 7)^2 - 7^2 + 9$

$\quad = (x - 7)^2 - 49 + 9$

$\boxed{S(x) = (x - 7)^2 - 40}$

$R(x) = x^2 + 7x - 2$

$\quad = \left(x + \dfrac{7}{2}\right)^2 - \left(\dfrac{7}{2}\right)^2 - 2$

$\quad = \left(x + \dfrac{7}{2}\right)^2 + \dfrac{-49}{4} - \dfrac{2 \times 4}{1 \times 4}$

$\quad = \left(x + \dfrac{7}{2}\right)^2 + \dfrac{-49}{4} - \dfrac{8}{4}$

$\boxed{R(x) = \left(x + \dfrac{7}{2}\right)^2 + \dfrac{-57}{4}}$

Corrigé de l'exercice 3

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = -2x^2 + 3x + 8$$
$$= -2 \times \left(x^2 - \frac{3}{2}x - 4\right)$$
$$= -2 \times \left(\left(x - \frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 - 4\right)$$
$$= -2 \times \left(\left(x - \frac{3}{4}\right)^2 + \frac{-9}{16} - \frac{4_{\times 16}}{1_{\times 16}}\right)$$
$$= -2 \times \left(\left(x - \frac{3}{4}\right)^2 + \frac{-9}{16} - \frac{64}{16}\right)$$
$$= -2 \times \left(\left(x - \frac{3}{4}\right)^2 + \frac{-73}{16}\right)$$
$$= -2 \times \left(x - \frac{3}{4}\right)^2 + \frac{-73 \times \cancel{2} \times (-1)}{\cancel{2} \times 8}$$
$$\boxed{P(x) = -2 \times \left(x - \frac{3}{4}\right)^2 + \frac{73}{8}}$$

$$Q(x) = x^2 - 7x + 8$$
$$= \left(x - \frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 8$$
$$= \left(x - \frac{7}{2}\right)^2 + \frac{-49}{4} + \frac{8_{\times 4}}{1_{\times 4}}$$
$$= \left(x - \frac{7}{2}\right)^2 + \frac{-49}{4} + \frac{32}{4}$$
$$\boxed{Q(x) = \left(x - \frac{7}{2}\right)^2 + \frac{-17}{4}}$$

$$R(x) = x^2 + 8x + 7$$
$$= (x+4)^2 - 4^2 + 7$$
$$= (x+4)^2 - 16 + 7$$
$$\boxed{R(x) = (x+4)^2 - 9}$$

$$S(x) = x^2 + 14x + 49$$
$$\boxed{S(x) = (x+7)^2}$$

Corrigé de l'exercice 4

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 + 4x + 4$$
$$\boxed{P(x) = (x+2)^2}$$

$$R(x) = x^2 + 18x + 6$$
$$= (x+9)^2 - 9^2 + 6$$
$$= (x+9)^2 - 81 + 6$$
$$\boxed{R(x) = (x+9)^2 - 75}$$

$$Q(x) = -4x^2 + 9x - 6$$
$$= -4 \times \left(x^2 - \frac{9}{4}x + \frac{3}{2}\right)$$
$$= -4 \times \left(\left(x - \frac{9}{8}\right)^2 - \left(\frac{9}{8}\right)^2 + \frac{3}{2}\right)$$
$$= -4 \times \left(\left(x - \frac{9}{8}\right)^2 + \frac{-81}{64} + \frac{3_{\times 32}}{2_{\times 32}}\right)$$
$$= -4 \times \left(\left(x - \frac{9}{8}\right)^2 + \frac{-81}{64} + \frac{96}{64}\right)$$
$$= -4 \times \left(\left(x - \frac{9}{8}\right)^2 + \frac{15}{64}\right)$$
$$= -4 \times \left(x - \frac{9}{8}\right)^2 + \frac{15 \times \cancel{4} \times (-1)}{\cancel{4} \times 16}$$
$$\boxed{Q(x) = -4 \times \left(x - \frac{9}{8}\right)^2 + \frac{-15}{16}}$$

$$S(x) = x^2 - 7x - 6$$
$$= \left(x - \frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 - 6$$
$$= \left(x - \frac{7}{2}\right)^2 + \frac{-49}{4} - \frac{6_{\times 4}}{1_{\times 4}}$$
$$= \left(x - \frac{7}{2}\right)^2 + \frac{-49}{4} - \frac{24}{4}$$
$$\boxed{S(x) = \left(x - \frac{7}{2}\right)^2 + \frac{-73}{4}}$$

Corrigé de l'exercice 5

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 4\,x^2 - 7\,x - 6$$

$$= 4 \times \left(x^2 - \frac{7}{4}\,x - \frac{3}{2}\right)$$

$$= 4 \times \left(\left(x - \frac{7}{8}\right)^2 - \left(\frac{7}{8}\right)^2 + \frac{-3}{2}\right)$$

$$= 4 \times \left(\left(x - \frac{7}{8}\right)^2 + \frac{-49}{64} + \frac{-3_{\times 32}}{2_{\times 32}}\right)$$

$$= 4 \times \left(\left(x - \frac{7}{8}\right)^2 + \frac{-49}{64} + \frac{-96}{64}\right)$$

$$= 4 \times \left(\left(x - \frac{7}{8}\right)^2 + \frac{-145}{64}\right)$$

$$= 4 \times \left(x - \frac{7}{8}\right)^2 + \frac{-145 \times \cancel{4}}{\cancel{4} \times 16}$$

$$\boxed{P(x) = 4 \times \left(x - \frac{7}{8}\right)^2 + \frac{-145}{16}}$$

$$Q(x) = x^2 + 18\,x - 2$$

$$= (x + 9)^2 - 9^2 - 2$$

$$= (x + 9)^2 - 81 - 2$$

$$\boxed{Q(x) = (x + 9)^2 - 83}$$

$$R(x) = 16\,x^2 + 32\,x + 16$$

$$= (4x + 4)^2$$

$$= \left(4 \times \left(x + \frac{4}{4}\right)\right)^2$$

$$\boxed{R(x) = 16 \times (x + 1)^2}$$

$$S(x) = x^2 - 11\,x - 4$$

$$= \left(x - \frac{11}{2}\right)^2 - \left(\frac{11}{2}\right)^2 - 4$$

$$= \left(x - \frac{11}{2}\right)^2 + \frac{-121}{4} - \frac{4_{\times 4}}{1_{\times 4}}$$

$$= \left(x - \frac{11}{2}\right)^2 + \frac{-121}{4} - \frac{16}{4}$$

$$\boxed{S(x) = \left(x - \frac{11}{2}\right)^2 + \frac{-137}{4}}$$

Corrigé de l'exercice 6

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 - 2\,x - 8$$

$$= (x - 1)^2 - 1^2 - 8$$

$$= (x - 1)^2 - 1 - 8$$

$$\boxed{P(x) = (x - 1)^2 - 9}$$

$$S(x) = 64\,x^2 + 112\,x + 49$$

$$= (8x + 7)^2$$

$$= \left(8 \times \left(x + \frac{7}{8}\right)\right)^2$$

$$\boxed{S(x) = 64 \times \left(x + \frac{7}{8}\right)^2}$$

$$Q(x) = 4\,x^2 - 4\,x + 7$$

$$= 4 \times \left(x^2 - x + \frac{7}{4}\right)$$

$$= 4 \times \left(\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + \frac{7}{4}\right)$$

$$= 4 \times \left(\left(x - \frac{1}{2}\right)^2 + \frac{6}{4}\right)$$

$$= 4 \times \left(\left(x - \frac{1}{2}\right)^2 + \frac{3 \times \cancel{2}}{\cancel{2} \times 2}\right)$$

$$= 4 \times \left(\left(x - \frac{1}{2}\right)^2 + \frac{3}{2}\right)$$

$$= 4 \times \left(x - \frac{1}{2}\right)^2 + \frac{3 \times \cancel{2} \times 2}{\cancel{2} \times 1}$$

$$\boxed{Q(x) = 4 \times \left(x - \frac{1}{2}\right)^2 + 6}$$

$$R(x) = x^2 - 5\,x - 6$$

$$= \left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 6$$

$$= \left(x - \frac{5}{2}\right)^2 + \frac{-25}{4} - \frac{6_{\times 4}}{1_{\times 4}}$$

$$= \left(x - \frac{5}{2}\right)^2 + \frac{-25}{4} - \frac{24}{4}$$

$$\boxed{R(x) = \left(x - \frac{5}{2}\right)^2 + \frac{-49}{4}}$$

Corrigé de l'exercice 7

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 + 7x - 9$$
$$= \left(x + \frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 - 9$$
$$= \left(x + \frac{7}{2}\right)^2 + \frac{-49}{4} - \frac{9_{\times 4}}{1_{\times 4}}$$
$$= \left(x + \frac{7}{2}\right)^2 + \frac{-49}{4} - \frac{36}{4}$$
$$\boxed{P(x) = \left(x + \frac{7}{2}\right)^2 + \frac{-85}{4}}$$

$$Q(x) = 16x^2 - 40x + 25$$
$$= (4x - 5)^2$$
$$= \left(4 \times \left(x - \frac{5}{4}\right)\right)^2$$
$$\boxed{Q(x) = 16 \times \left(x - \frac{5}{4}\right)^2}$$

$$S(x) = x^2 - 14x - 1$$
$$= (x - 7)^2 - 7^2 - 1$$
$$= (x - 7)^2 - 49 - 1$$
$$\boxed{S(x) = (x - 7)^2 - 50}$$

$$R(x) = -4x^2 - 9x - 6$$
$$= -4 \times \left(x^2 + \frac{9}{4}x + \frac{3}{2}\right)$$
$$= -4 \times \left(\left(x + \frac{9}{8}\right)^2 - \left(\frac{9}{8}\right)^2 + \frac{3}{2}\right)$$
$$= -4 \times \left(\left(x + \frac{9}{8}\right)^2 + \frac{-81}{64} + \frac{3_{\times 32}}{2_{\times 32}}\right)$$
$$= -4 \times \left(\left(x + \frac{9}{8}\right)^2 + \frac{-81}{64} + \frac{96}{64}\right)$$
$$= -4 \times \left(\left(x + \frac{9}{8}\right)^2 + \frac{15}{64}\right)$$
$$= -4 \times \left(x + \frac{9}{8}\right)^2 + \frac{15 \times \cancel{4} \times (-1)}{\cancel{4} \times 16}$$
$$\boxed{R(x) = -4 \times \left(x + \frac{9}{8}\right)^2 + \frac{-15}{16}}$$

Corrigé de l'exercice 8

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 + 12x - 9$$
$$= (x + 6)^2 - 6^2 - 9$$
$$= (x + 6)^2 - 36 - 9$$
$$\boxed{P(x) = (x + 6)^2 - 45}$$

$$S(x) = 81x^2 - 72x + 16$$
$$= (9x - 4)^2$$
$$= \left(9 \times \left(x - \frac{4}{9}\right)\right)^2$$
$$\boxed{S(x) = 81 \times \left(x - \frac{4}{9}\right)^2}$$

$$Q(x) = 4x^2 + 9x + 8$$
$$= 4 \times \left(x^2 + \frac{9}{4}x + 2\right)$$
$$= 4 \times \left(\left(x + \frac{9}{8}\right)^2 - \left(\frac{9}{8}\right)^2 + 2\right)$$
$$= 4 \times \left(\left(x + \frac{9}{8}\right)^2 + \frac{-81}{64} + \frac{2_{\times 64}}{1_{\times 64}}\right)$$
$$= 4 \times \left(\left(x + \frac{9}{8}\right)^2 + \frac{-81}{64} + \frac{128}{64}\right)$$
$$= 4 \times \left(\left(x + \frac{9}{8}\right)^2 + \frac{47}{64}\right)$$
$$= 4 \times \left(x + \frac{9}{8}\right)^2 + \frac{47 \times \cancel{4}}{\cancel{4} \times 16}$$
$$\boxed{Q(x) = 4 \times \left(x + \frac{9}{8}\right)^2 + \frac{47}{16}}$$

$$R(x) = x^2 + 5x - 3$$
$$= \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 3$$
$$= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} - \frac{3_{\times 4}}{1_{\times 4}}$$
$$= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} - \frac{12}{4}$$
$$\boxed{R(x) = \left(x + \frac{5}{2}\right)^2 + \frac{-37}{4}}$$