Corrigé de l'exercice 1

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 9\,x^2 - 48\,x + 64$$
$$= (3\,x - 8)^2$$
$$= \left(3 \times \left(x - \frac{8}{3}\right)\right)^2$$
$$\boxed{P(x) = 9 \times \left(x - \frac{8}{3}\right)^2}$$

$$Q(x) = x^2 + 16\,x - 1$$
$$= (x + 8)^2 - 8^2 - 1$$
$$= (x + 8)^2 - 64 - 1$$
$$\boxed{Q(x) = (x + 8)^2 - 65}$$

$$R(x) = x^2 + 9\,x - 8$$
$$= \left(x + \frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 - 8$$
$$= \left(x + \frac{9}{2}\right)^2 + \frac{-81}{4} - \frac{8_{\times 4}}{1_{\times 4}}$$
$$= \left(x + \frac{9}{2}\right)^2 + \frac{-81}{4} - \frac{32}{4}$$
$$\boxed{R(x) = \left(x + \frac{9}{2}\right)^2 + \frac{-113}{4}}$$

$$S(x) = -5\,x^2 - 9\,x - 3$$
$$= -5 \times \left(x^2 + \frac{9}{5}\,x + \frac{3}{5}\right)$$
$$= -5 \times \left(\left(x + \frac{9}{10}\right)^2 - \left(\frac{9}{10}\right)^2 + \frac{3}{5}\right)$$
$$= -5 \times \left(\left(x + \frac{9}{10}\right)^2 + \frac{-81}{100} + \frac{3_{\times 20}}{5_{\times 20}}\right)$$
$$= -5 \times \left(\left(x + \frac{9}{10}\right)^2 + \frac{-81}{100} + \frac{60}{100}\right)$$
$$= -5 \times \left(\left(x + \frac{9}{10}\right)^2 + \frac{-21}{100}\right)$$
$$= -5 \times \left(x + \frac{9}{10}\right)^2 + \frac{-21 \times \cancel{5} \times (-1)}{\cancel{5} \times 20}$$
$$\boxed{S(x) = -5 \times \left(x + \frac{9}{10}\right)^2 + \frac{21}{20}}$$

Corrigé de l'exercice 2

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 5\,x^2 + x - 7$$
$$= 5 \times \left(x^2 + \frac{1}{5}\,x - \frac{7}{5}\right)$$
$$= 5 \times \left(\left(x + \frac{1}{10}\right)^2 - \left(\frac{1}{10}\right)^2 + \frac{-7}{5}\right)$$
$$= 5 \times \left(\left(x + \frac{1}{10}\right)^2 + \frac{-1}{100} + \frac{-7_{\times 20}}{5_{\times 20}}\right)$$
$$= 5 \times \left(\left(x + \frac{1}{10}\right)^2 + \frac{-1}{100} + \frac{-140}{100}\right)$$
$$= 5 \times \left(\left(x + \frac{1}{10}\right)^2 + \frac{-141}{100}\right)$$
$$= 5 \times \left(x + \frac{1}{10}\right)^2 + \frac{-141 \times \cancel{5}}{\cancel{5} \times 20}$$
$$\boxed{P(x) = 5 \times \left(x + \frac{1}{10}\right)^2 + \frac{-141}{20}}$$

$$Q(x) = x^2 + 11\,x + 8$$
$$= \left(x + \frac{11}{2}\right)^2 - \left(\frac{11}{2}\right)^2 + 8$$
$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{8_{\times 4}}{1_{\times 4}}$$
$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{32}{4}$$
$$\boxed{Q(x) = \left(x + \frac{11}{2}\right)^2 + \frac{-89}{4}}$$

$$R(x) = x^2 - 12\,x + 6$$
$$= (x - 6)^2 - 6^2 + 6$$
$$= (x - 6)^2 - 36 + 6$$
$$\boxed{R(x) = (x - 6)^2 - 30}$$

$$S(x) = 9\,x^2 + 42\,x + 49$$
$$= (3\,x + 7)^2$$
$$= \left(3 \times \left(x + \frac{7}{3}\right)\right)^2$$
$$\boxed{S(x) = 9 \times \left(x + \frac{7}{3}\right)^2}$$

Corrigé de l'exercice 3

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = -2\,x^2 - x - 5$$
$$= -2 \times \left(x^2 + \frac{1}{2}\,x + \frac{5}{2}\right)$$
$$= -2 \times \left(\left(x + \frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 + \frac{5}{2}\right)$$
$$= -2 \times \left(\left(x + \frac{1}{4}\right)^2 + \frac{-1}{16} + \frac{5_{\times 8}}{2_{\times 8}}\right)$$
$$= -2 \times \left(\left(x + \frac{1}{4}\right)^2 + \frac{-1}{16} + \frac{40}{16}\right)$$
$$= -2 \times \left(\left(x + \frac{1}{4}\right)^2 + \frac{39}{16}\right)$$
$$= -2 \times \left(x + \frac{1}{4}\right)^2 + \frac{39 \times \cancel{2} \times (-1)}{\cancel{2} \times 8}$$
$$\boxed{P(x) = -2 \times \left(x + \frac{1}{4}\right)^2 + \frac{-39}{8}}$$

$$Q(x) = 64\,x^2 - 128\,x + 64$$
$$= (8\,x - 8)^2$$
$$= \left(8 \times \left(x - \frac{8}{8}\right)\right)^2$$
$$\boxed{Q(x) = 64 \times (x - 1)^2}$$

$$R(x) = x^2 - 16\,x + 4$$
$$= (x - 8)^2 - 8^2 + 4$$
$$= (x - 8)^2 - 64 + 4$$
$$\boxed{R(x) = (x - 8)^2 - 60}$$

$$S(x) = x^2 + 11\,x + 3$$
$$= \left(x + \frac{11}{2}\right)^2 - \left(\frac{11}{2}\right)^2 + 3$$
$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{3_{\times 4}}{1_{\times 4}}$$
$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{12}{4}$$
$$\boxed{S(x) = \left(x + \frac{11}{2}\right)^2 + \frac{-109}{4}}$$

Corrigé de l'exercice 4

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 + 5\,x + 2$$
$$= \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 2$$
$$= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{2_{\times 4}}{1_{\times 4}}$$
$$= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{8}{4}$$
$$\boxed{P(x) = \left(x + \frac{5}{2}\right)^2 + \frac{-17}{4}}$$

$$Q(x) = x^2 - 2\,x - 3$$
$$= (x - 1)^2 - 1^2 - 3$$
$$= (x - 1)^2 - 1 - 3$$
$$\boxed{Q(x) = (x - 1)^2 - 4}$$

$$R(x) = 16\,x^2 - 16\,x + 4$$
$$= (4\,x - 2)^2$$
$$= \left(4 \times \left(x - \frac{2}{4}\right)\right)^2$$
$$\boxed{R(x) = 16 \times \left(x - \frac{1}{2}\right)^2}$$

$$S(x) = 5\,x^2 + 9\,x + 5$$
$$= 5 \times \left(x^2 + \frac{9}{5}\,x + 1\right)$$
$$= 5 \times \left(\left(x + \frac{9}{10}\right)^2 - \left(\frac{9}{10}\right)^2 + 1\right)$$
$$= 5 \times \left(\left(x + \frac{9}{10}\right)^2 + \frac{-81}{100} + \frac{1_{\times 100}}{1_{\times 100}}\right)$$
$$= 5 \times \left(\left(x + \frac{9}{10}\right)^2 + \frac{-81}{100} + \frac{100}{100}\right)$$
$$= 5 \times \left(\left(x + \frac{9}{10}\right)^2 + \frac{19}{100}\right)$$
$$= 5 \times \left(x + \frac{9}{10}\right)^2 + \frac{19 \times \cancel{5}}{\cancel{5} \times 20}$$
$$\boxed{S(x) = 5 \times \left(x + \frac{9}{10}\right)^2 + \frac{19}{20}}$$

Corrigé de l'exercice 5

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 - 10x + 1$$
$$= (x-5)^2 - 5^2 + 1$$
$$= (x-5)^2 - 25 + 1$$
$$\boxed{P(x) = (x-5)^2 - 24}$$

$$R(x) = 9x^2 + 30x + 25$$
$$= (3x+5)^2$$
$$= \left(3 \times \left(x + \frac{5}{3}\right)\right)^2$$
$$\boxed{R(x) = 9 \times \left(x + \frac{5}{3}\right)^2}$$

$$Q(x) = x^2 + 11x - 1$$
$$= \left(x + \frac{11}{2}\right)^2 - \left(\frac{11}{2}\right)^2 - 1$$
$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} - \frac{1_{\times 4}}{1_{\times 4}}$$
$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} - \frac{4}{4}$$
$$\boxed{Q(x) = \left(x + \frac{11}{2}\right)^2 + \frac{-125}{4}}$$

$$S(x) = -5x^2 - 5x - 2$$
$$= -5 \times \left(x^2 + x + \frac{2}{5}\right)$$
$$= -5 \times \left(\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + \frac{2}{5}\right)$$
$$= -5 \times \left(\left(x + \frac{1}{2}\right)^2 + \frac{-1_{\times 5}}{4_{\times 5}} + \frac{2_{\times 4}}{5_{\times 4}}\right)$$
$$= -5 \times \left(\left(x + \frac{1}{2}\right)^2 + \frac{-5}{20} + \frac{8}{20}\right)$$
$$= -5 \times \left(\left(x + \frac{1}{2}\right)^2 + \frac{3}{20}\right)$$
$$= -5 \times \left(x + \frac{1}{2}\right)^2 + \frac{3 \times \cancel{5} \times (-1)}{\cancel{5} \times 4}$$
$$\boxed{S(x) = -5 \times \left(x + \frac{1}{2}\right)^2 + \frac{-3}{4}}$$

Corrigé de l'exercice 6

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 - 5x + 7$$
$$= \left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 7$$
$$= \left(x - \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{7_{\times 4}}{1_{\times 4}}$$
$$= \left(x - \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{28}{4}$$
$$\boxed{P(x) = \left(x - \frac{5}{2}\right)^2 + \frac{3}{4}}$$

$$Q(x) = x^2 + 10x + 1$$
$$= (x+5)^2 - 5^2 + 1$$
$$= (x+5)^2 - 25 + 1$$
$$\boxed{Q(x) = (x+5)^2 - 24}$$

$$R(x) = 49x^2 - 126x + 81$$
$$= (7x - 9)^2$$
$$= \left(7 \times \left(x - \frac{9}{7}\right)\right)^2$$
$$\boxed{R(x) = 49 \times \left(x - \frac{9}{7}\right)^2}$$

$$S(x) = -4x^2 - 6x - 2$$
$$= -4 \times \left(x^2 + \frac{3}{2}x + \frac{1}{2}\right)$$
$$= -4 \times \left(\left(x + \frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 + \frac{1}{2}\right)$$
$$= -4 \times \left(\left(x + \frac{3}{4}\right)^2 + \frac{-9}{16} + \frac{1_{\times 8}}{2_{\times 8}}\right)$$
$$= -4 \times \left(\left(x + \frac{3}{4}\right)^2 + \frac{-9}{16} + \frac{8}{16}\right)$$
$$= -4 \times \left(\left(x + \frac{3}{4}\right)^2 + \frac{-1}{16}\right)$$
$$= -4 \times \left(x + \frac{3}{4}\right)^2 + \frac{-1 \times \cancel{4} \times (-1) \times 1}{\cancel{4} \times 4}$$
$$\boxed{S(x) = -4 \times \left(x + \frac{3}{4}\right)^2 + \frac{1}{4}}$$

Corrigé de l'exercice 7

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 + 4x - 1$$
$$= (x+2)^2 - 2^2 - 1$$
$$= (x+2)^2 - 4 - 1$$
$$\boxed{P(x) = (x+2)^2 - 5}$$

$$R(x) = 49x^2 + 56x + 16$$
$$= (7x+4)^2$$
$$= \left(7 \times \left(x + \frac{4}{7}\right)\right)^2$$
$$\boxed{R(x) = 49 \times \left(x + \frac{4}{7}\right)^2}$$

$$Q(x) = 3x^2 - 5x + 5$$
$$= 3 \times \left(x^2 - \frac{5}{3}x + \frac{5}{3}\right)$$
$$= 3 \times \left(\left(x - \frac{5}{6}\right)^2 - \left(\frac{5}{6}\right)^2 + \frac{5}{3}\right)$$
$$= 3 \times \left(\left(x - \frac{5}{6}\right)^2 + \frac{-25}{36} + \frac{5_{\times 12}}{3_{\times 12}}\right)$$
$$= 3 \times \left(\left(x - \frac{5}{6}\right)^2 + \frac{-25}{36} + \frac{60}{36}\right)$$
$$= 3 \times \left(\left(x - \frac{5}{6}\right)^2 + \frac{35}{36}\right)$$
$$= 3 \times \left(x - \frac{5}{6}\right)^2 + \frac{35 \times \cancel{3}}{\cancel{3} \times 12}$$
$$\boxed{Q(x) = 3 \times \left(x - \frac{5}{6}\right)^2 + \frac{35}{12}}$$

$$S(x) = x^2 - 5x + 4$$
$$= \left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 4$$
$$= \left(x - \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{4_{\times 4}}{1_{\times 4}}$$
$$= \left(x - \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{16}{4}$$
$$\boxed{S(x) = \left(x - \frac{5}{2}\right)^2 + \frac{-9}{4}}$$

Corrigé de l'exercice 8

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 3x^2 - 5x + 4$$
$$= 3 \times \left(x^2 - \frac{5}{3}x + \frac{4}{3}\right)$$
$$= 3 \times \left(\left(x - \frac{5}{6}\right)^2 - \left(\frac{5}{6}\right)^2 + \frac{4}{3}\right)$$
$$= 3 \times \left(\left(x - \frac{5}{6}\right)^2 + \frac{-25}{36} + \frac{4_{\times 12}}{3_{\times 12}}\right)$$
$$= 3 \times \left(\left(x - \frac{5}{6}\right)^2 + \frac{-25}{36} + \frac{48}{36}\right)$$
$$= 3 \times \left(\left(x - \frac{5}{6}\right)^2 + \frac{23}{36}\right)$$
$$= 3 \times \left(x - \frac{5}{6}\right)^2 + \frac{23 \times \cancel{3}}{\cancel{3} \times 12}$$
$$\boxed{P(x) = 3 \times \left(x - \frac{5}{6}\right)^2 + \frac{23}{12}}$$

$$Q(x) = x^2 + 10x - 1$$
$$= (x+5)^2 - 5^2 - 1$$
$$= (x+5)^2 - 25 - 1$$
$$\boxed{Q(x) = (x+5)^2 - 26}$$

$$R(x) = 25x^2 + 30x + 9$$
$$= (5x+3)^2$$
$$= \left(5 \times \left(x + \frac{3}{5}\right)\right)^2$$
$$\boxed{R(x) = 25 \times \left(x + \frac{3}{5}\right)^2}$$

$$S(x) = x^2 + 5x + 3$$
$$= \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 3$$
$$= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{3_{\times 4}}{1_{\times 4}}$$
$$= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{12}{4}$$
$$\boxed{S(x) = \left(x + \frac{5}{2}\right)^2 + \frac{-13}{4}}$$