

Corrigé de l'exercice 1Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= 36x^2 - 84x + 49 \\ &= (6x - 7)^2 \\ &= \left(6 \times \left(x - \frac{7}{6}\right)\right)^2 \end{aligned}$$

$$P(x) = 36 \times \left(x - \frac{7}{6}\right)^2$$

$$\begin{aligned} S(x) &= x^2 - 8x - 4 \\ &= (x - 4)^2 - 4^2 - 4 \\ &= (x - 4)^2 - 16 - 4 \end{aligned}$$

$$S(x) = (x - 4)^2 - 20$$

$$\begin{aligned} Q(x) &= x^2 + 3x + 3 \\ &= \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 3 \\ &= \left(x + \frac{3}{2}\right)^2 + \frac{-9}{4} + \frac{3 \times 4}{1 \times 4} \\ &= \left(x + \frac{3}{2}\right)^2 + \frac{-9}{4} + \frac{12}{4} \end{aligned}$$

$$Q(x) = \left(x + \frac{3}{2}\right)^2 + \frac{3}{4}$$

$$\begin{aligned} R(x) &= -5x^2 + 4x - 4 \\ &= -5 \times \left(x^2 - \frac{4}{5}x + \frac{4}{5}\right) \\ &= -5 \times \left(\left(x - \frac{2}{5}\right)^2 - \left(\frac{2}{5}\right)^2 + \frac{4}{5}\right) \\ &= -5 \times \left(\left(x - \frac{2}{5}\right)^2 + \frac{-4}{25} + \frac{4 \times 5}{5 \times 5}\right) \\ &= -5 \times \left(\left(x - \frac{2}{5}\right)^2 + \frac{-4}{25} + \frac{20}{25}\right) \\ &= -5 \times \left(\left(x - \frac{2}{5}\right)^2 + \frac{16}{25}\right) \\ &= -5 \times \left(x - \frac{2}{5}\right)^2 + \frac{16 \times 5 \times (-1)}{5 \times 5} \end{aligned}$$

$$R(x) = -5 \times \left(x - \frac{2}{5}\right)^2 + \frac{-16}{5}$$

Corrigé de l'exercice 2Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= 2x^2 - 9x - 1 \\ &= 2 \times \left(x^2 - \frac{9}{2}x - \frac{1}{2}\right) \\ &= 2 \times \left(\left(x - \frac{9}{4}\right)^2 - \left(\frac{9}{4}\right)^2 + \frac{-1}{2}\right) \\ &= 2 \times \left(\left(x - \frac{9}{4}\right)^2 + \frac{-81}{16} + \frac{-1 \times 8}{2 \times 8}\right) \\ &= 2 \times \left(\left(x - \frac{9}{4}\right)^2 + \frac{-81}{16} + \frac{-8}{16}\right) \\ &= 2 \times \left(\left(x - \frac{9}{4}\right)^2 + \frac{-89}{16}\right) \\ &= 2 \times \left(x - \frac{9}{4}\right)^2 + \frac{-89 \times 2}{2 \times 8} \end{aligned}$$

$$P(x) = 2 \times \left(x - \frac{9}{4}\right)^2 + \frac{-89}{8}$$

$$\begin{aligned} Q(x) &= x^2 - 11x + 1 \\ &= \left(x - \frac{11}{2}\right)^2 - \left(\frac{11}{2}\right)^2 + 1 \\ &= \left(x - \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{1 \times 4}{1 \times 4} \\ &= \left(x - \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{4}{4} \end{aligned}$$

$$Q(x) = \left(x - \frac{11}{2}\right)^2 + \frac{-117}{4}$$

$$\begin{aligned} R(x) &= x^2 + 10x - 7 \\ &= (x + 5)^2 - 5^2 - 7 \\ &= (x + 5)^2 - 25 - 7 \end{aligned}$$

$$R(x) = (x + 5)^2 - 32$$

$$S(x) = x^2 + 16x + 64$$

$$S(x) = (x + 8)^2$$

Corrigé de l'exercice 3

Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= x^2 - 12x + 3 \\ &= (x - 6)^2 - 6^2 + 3 \\ &= (x - 6)^2 - 36 + 3 \end{aligned}$$

$$\boxed{P(x) = (x - 6)^2 - 33}$$

$$\begin{aligned} R(x) &= 9x^2 - 18x + 9 \\ &= (3x - 3)^2 \\ &= \left(3 \times \left(x - \frac{3}{3}\right)\right)^2 \end{aligned}$$

$$\boxed{R(x) = 9 \times (x - 1)^2}$$

$$\begin{aligned} Q(x) &= x^2 - 9x - 4 \\ &= \left(x - \frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 - 4 \\ &= \left(x - \frac{9}{2}\right)^2 + \frac{-81}{4} - \frac{4 \times 4}{1 \times 4} \\ &= \left(x - \frac{9}{2}\right)^2 + \frac{-81}{4} - \frac{16}{4} \end{aligned}$$

$$\boxed{Q(x) = \left(x - \frac{9}{2}\right)^2 + \frac{-97}{4}}$$

$$\begin{aligned} S(x) &= 5x^2 + 6x - 7 \\ &= 5 \times \left(x^2 + \frac{6}{5}x - \frac{7}{5}\right) \\ &= 5 \times \left(\left(x + \frac{3}{5}\right)^2 - \left(\frac{3}{5}\right)^2 + \frac{-7}{5}\right) \\ &= 5 \times \left(\left(x + \frac{3}{5}\right)^2 + \frac{-9}{25} + \frac{-7 \times 5}{5 \times 5}\right) \\ &= 5 \times \left(\left(x + \frac{3}{5}\right)^2 + \frac{-9}{25} + \frac{-35}{25}\right) \\ &= 5 \times \left(\left(x + \frac{3}{5}\right)^2 + \frac{-44}{25}\right) \\ &= 5 \times \left(x + \frac{3}{5}\right)^2 + \frac{-44 \times 5}{5 \times 5} \end{aligned}$$

$$\boxed{S(x) = 5 \times \left(x + \frac{3}{5}\right)^2 + \frac{-44}{5}}$$

Corrigé de l'exercice 4

Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= 2x^2 + 4x - 6 \\ &= 2 \times (x^2 + 2x - 3) \\ &= 2 \times \left((x + 1)^2 - 1^2 - 3\right) \\ &= 2 \times \left((x + 1)^2 - 1 - 3\right) \\ &= 2 \times \left((x + 1)^2 - 4\right) \end{aligned}$$

$$\boxed{P(x) = 2 \times (x + 1)^2 - 8}$$

$$\begin{aligned} Q(x) &= x^2 - 6x - 1 \\ &= (x - 3)^2 - 3^2 - 1 \\ &= (x - 3)^2 - 9 - 1 \end{aligned}$$

$$\boxed{Q(x) = (x - 3)^2 - 10}$$

$$\begin{aligned} R(x) &= 49x^2 - 28x + 4 \\ &= (7x - 2)^2 \\ &= \left(7 \times \left(x - \frac{2}{7}\right)\right)^2 \end{aligned}$$

$$\boxed{R(x) = 49 \times \left(x - \frac{2}{7}\right)^2}$$

$$\begin{aligned} S(x) &= x^2 + 9x - 5 \\ &= \left(x + \frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 - 5 \\ &= \left(x + \frac{9}{2}\right)^2 + \frac{-81}{4} - \frac{5 \times 4}{1 \times 4} \\ &= \left(x + \frac{9}{2}\right)^2 + \frac{-81}{4} - \frac{20}{4} \end{aligned}$$

$$\boxed{S(x) = \left(x + \frac{9}{2}\right)^2 + \frac{-101}{4}}$$

Corrigé de l'exercice 5

Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= x^2 - 6x + 8 \\ &= (x - 3)^2 - 3^2 + 8 \\ &= (x - 3)^2 - 9 + 8 \end{aligned}$$

$$\boxed{P(x) = (x - 3)^2 - 1}$$

$$R(x) = x^2 + 16x + 64$$

$$\boxed{R(x) = (x + 8)^2}$$

$$\begin{aligned} Q(x) &= 5x^2 + 4x - 6 \\ &= 5 \times \left(x^2 + \frac{4}{5}x - \frac{6}{5} \right) \\ &= 5 \times \left(\left(x + \frac{2}{5} \right)^2 - \left(\frac{2}{5} \right)^2 + \frac{-6}{5} \right) \\ &= 5 \times \left(\left(x + \frac{2}{5} \right)^2 + \frac{-4}{25} + \frac{-6 \times 5}{5 \times 5} \right) \\ &= 5 \times \left(\left(x + \frac{2}{5} \right)^2 + \frac{-4}{25} + \frac{-30}{25} \right) \\ &= 5 \times \left(\left(x + \frac{2}{5} \right)^2 + \frac{-34}{25} \right) \\ &= 5 \times \left(x + \frac{2}{5} \right)^2 + \frac{-34 \times 5}{5 \times 5} \end{aligned}$$

$$\boxed{Q(x) = 5 \times \left(x + \frac{2}{5} \right)^2 + \frac{-34}{5}}$$

$$\begin{aligned} S(x) &= x^2 - 7x + 9 \\ &= \left(x - \frac{7}{2} \right)^2 - \left(\frac{7}{2} \right)^2 + 9 \\ &= \left(x - \frac{7}{2} \right)^2 + \frac{-49}{4} + \frac{9 \times 4}{1 \times 4} \\ &= \left(x - \frac{7}{2} \right)^2 + \frac{-49}{4} + \frac{36}{4} \end{aligned}$$

$$\boxed{S(x) = \left(x - \frac{7}{2} \right)^2 + \frac{-13}{4}}$$

Corrigé de l'exercice 6

Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= x^2 - 3x + 2 \\ &= \left(x - \frac{3}{2} \right)^2 - \left(\frac{3}{2} \right)^2 + 2 \\ &= \left(x - \frac{3}{2} \right)^2 + \frac{-9}{4} + \frac{2 \times 4}{1 \times 4} \\ &= \left(x - \frac{3}{2} \right)^2 + \frac{-9}{4} + \frac{8}{4} \end{aligned}$$

$$\boxed{P(x) = \left(x - \frac{3}{2} \right)^2 + \frac{-1}{4}}$$

$$\begin{aligned} Q(x) &= -3x^2 - 2x + 8 \\ &= -3 \times \left(x^2 + \frac{2}{3}x - \frac{8}{3} \right) \\ &= -3 \times \left(\left(x + \frac{1}{3} \right)^2 - \left(\frac{1}{3} \right)^2 + \frac{-8}{3} \right) \\ &= -3 \times \left(\left(x + \frac{1}{3} \right)^2 + \frac{-1}{9} + \frac{-8 \times 3}{3 \times 3} \right) \\ &= -3 \times \left(\left(x + \frac{1}{3} \right)^2 + \frac{-1}{9} + \frac{-24}{9} \right) \\ &= -3 \times \left(\left(x + \frac{1}{3} \right)^2 + \frac{-25}{9} \right) \\ &= -3 \times \left(x + \frac{1}{3} \right)^2 + \frac{-25 \times 3 \times (-1)}{3 \times 3} \end{aligned}$$

$$\boxed{Q(x) = -3 \times \left(x + \frac{1}{3} \right)^2 + \frac{25}{3}}$$

$$\begin{aligned} R(x) &= x^2 - 2x - 9 \\ &= (x - 1)^2 - 1^2 - 9 \\ &= (x - 1)^2 - 1 - 9 \end{aligned}$$

$$\boxed{R(x) = (x - 1)^2 - 10}$$

$$\begin{aligned} S(x) &= 16x^2 + 72x + 81 \\ &= (4x + 9)^2 \\ &= \left(4 \times \left(x + \frac{9}{4} \right) \right)^2 \end{aligned}$$

$$\boxed{S(x) = 16 \times \left(x + \frac{9}{4} \right)^2}$$

Corrigé de l'exercice 7

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 4x^2 + 36x + 81$$

$$= (2x + 9)^2$$

$$= \left(2 \times \left(x + \frac{9}{2}\right)\right)^2$$

$$\boxed{P(x) = 4 \times \left(x + \frac{9}{2}\right)^2}$$

$$Q(x) = x^2 + 8x + 5$$

$$= (x + 4)^2 - 4^2 + 5$$

$$= (x + 4)^2 - 16 + 5$$

$$\boxed{Q(x) = (x + 4)^2 - 11}$$

$$R(x) = x^2 + 11x + 5$$

$$= \left(x + \frac{11}{2}\right)^2 - \left(\frac{11}{2}\right)^2 + 5$$

$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{5 \times 4}{1 \times 4}$$

$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{20}{4}$$

$$\boxed{R(x) = \left(x + \frac{11}{2}\right)^2 + \frac{-101}{4}}$$

$$S(x) = 2x^2 + 7x - 7$$

$$= 2 \times \left(x^2 + \frac{7}{2}x - \frac{7}{2}\right)$$

$$= 2 \times \left(\left(x + \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{-7}{2}\right)$$

$$= 2 \times \left(\left(x + \frac{7}{4}\right)^2 + \frac{-49}{16} + \frac{-7 \times 8}{2 \times 8}\right)$$

$$= 2 \times \left(\left(x + \frac{7}{4}\right)^2 + \frac{-49}{16} + \frac{-56}{16}\right)$$

$$= 2 \times \left(\left(x + \frac{7}{4}\right)^2 + \frac{-105}{16}\right)$$

$$= 2 \times \left(x + \frac{7}{4}\right)^2 + \frac{-105 \times 2}{2 \times 8}$$

$$\boxed{S(x) = 2 \times \left(x + \frac{7}{4}\right)^2 + \frac{-105}{8}}$$

Corrigé de l'exercice 8

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 + 8x - 9$$

$$= (x + 4)^2 - 4^2 - 9$$

$$= (x + 4)^2 - 16 - 9$$

$$\boxed{P(x) = (x + 4)^2 - 25}$$

$$Q(x) = x^2 + 3x - 3$$

$$= \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 3$$

$$= \left(x + \frac{3}{2}\right)^2 + \frac{-9}{4} - \frac{3 \times 4}{1 \times 4}$$

$$= \left(x + \frac{3}{2}\right)^2 + \frac{-9}{4} - \frac{12}{4}$$

$$\boxed{Q(x) = \left(x + \frac{3}{2}\right)^2 + \frac{-21}{4}}$$

$$R(x) = 81x^2 + 162x + 81$$

$$= (9x + 9)^2$$

$$= \left(9 \times \left(x + \frac{9}{9}\right)\right)^2$$

$$\boxed{R(x) = 81 \times (x + 1)^2}$$

$$S(x) = 3x^2 + 7x + 6$$

$$= 3 \times \left(x^2 + \frac{7}{3}x + 2\right)$$

$$= 3 \times \left(\left(x + \frac{7}{6}\right)^2 - \left(\frac{7}{6}\right)^2 + 2\right)$$

$$= 3 \times \left(\left(x + \frac{7}{6}\right)^2 + \frac{-49}{36} + \frac{2 \times 36}{1 \times 36}\right)$$

$$= 3 \times \left(\left(x + \frac{7}{6}\right)^2 + \frac{-49}{36} + \frac{72}{36}\right)$$

$$= 3 \times \left(\left(x + \frac{7}{6}\right)^2 + \frac{23}{36}\right)$$

$$= 3 \times \left(x + \frac{7}{6}\right)^2 + \frac{23 \times 3}{3 \times 12}$$

$$\boxed{S(x) = 3 \times \left(x + \frac{7}{6}\right)^2 + \frac{23}{12}}$$