

Corrigé de l'exercice 1Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= 25x^2 + 60x + 36 \\ &= (5x + 6)^2 \\ &= \left(5 \times \left(x + \frac{6}{5}\right)\right)^2 \end{aligned}$$

$$P(x) = 25 \times \left(x + \frac{6}{5}\right)^2$$

$$\begin{aligned} Q(x) &= x^2 - 16x + 9 \\ &= (x - 8)^2 - 8^2 + 9 \\ &= (x - 8)^2 - 64 + 9 \end{aligned}$$

$$Q(x) = (x - 8)^2 - 55$$

$$\begin{aligned} R(x) &= 4x^2 + 4x + 8 \\ &= 4 \times (x^2 + x + 2) \\ &= 4 \times \left(\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 2\right) \\ &= 4 \times \left(\left(x + \frac{1}{2}\right)^2 + \frac{-1}{4} + \frac{2 \times 4}{1 \times 4}\right) \end{aligned}$$

$$\begin{aligned} &= 4 \times \left(\left(x + \frac{1}{2}\right)^2 + \frac{-1}{4} + \frac{8}{4}\right) \\ &= 4 \times \left(\left(x + \frac{1}{2}\right)^2 + \frac{7}{4}\right) \\ &= 4 \times \left(x + \frac{1}{2}\right)^2 + \frac{7 \times 4}{4 \times 1} \end{aligned}$$

$$R(x) = 4 \times \left(x + \frac{1}{2}\right)^2 + 7$$

$$\begin{aligned} S(x) &= x^2 + 11x - 1 \\ &= \left(x + \frac{11}{2}\right)^2 - \left(\frac{11}{2}\right)^2 - 1 \\ &= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} - \frac{1 \times 4}{1 \times 4} \\ &= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} - \frac{4}{4} \end{aligned}$$

$$S(x) = \left(x + \frac{11}{2}\right)^2 + \frac{-125}{4}$$

Corrigé de l'exercice 2Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= x^2 + 12x + 1 \\ &= (x + 6)^2 - 6^2 + 1 \\ &= (x + 6)^2 - 36 + 1 \end{aligned}$$

$$P(x) = (x + 6)^2 - 35$$

$$R(x) = x^2 + 6x + 9$$

$$R(x) = (x + 3)^2$$

$$\begin{aligned} Q(x) &= 3x^2 + 8x - 8 \\ &= 3 \times \left(x^2 + \frac{8}{3}x - \frac{8}{3}\right) \\ &= 3 \times \left(\left(x + \frac{4}{3}\right)^2 - \left(\frac{4}{3}\right)^2 + \frac{-8}{3}\right) \\ &= 3 \times \left(\left(x + \frac{4}{3}\right)^2 + \frac{-16}{9} + \frac{-8 \times 3}{3 \times 3}\right) \\ &= 3 \times \left(\left(x + \frac{4}{3}\right)^2 + \frac{-16}{9} + \frac{-24}{9}\right) \\ &= 3 \times \left(\left(x + \frac{4}{3}\right)^2 + \frac{-40}{9}\right) \\ &= 3 \times \left(x + \frac{4}{3}\right)^2 + \frac{-40 \times 3}{3 \times 3} \end{aligned}$$

$$Q(x) = 3 \times \left(x + \frac{4}{3}\right)^2 + \frac{-40}{3}$$

$$\begin{aligned} S(x) &= x^2 - 5x + 9 \\ &= \left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 9 \\ &= \left(x - \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{9 \times 4}{1 \times 4} \\ &= \left(x - \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{36}{4} \end{aligned}$$

$$S(x) = \left(x - \frac{5}{2}\right)^2 + \frac{11}{4}$$

Corrigé de l'exercice 3

Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= x^2 - 2x - 3 \\ &= (x-1)^2 - 1^2 - 3 \\ &= (x-1)^2 - 1 - 3 \end{aligned}$$

$$\boxed{P(x) = (x-1)^2 - 4}$$

$$\begin{aligned} Q(x) &= 81x^2 + 72x + 16 \\ &= (9x+4)^2 \\ &= \left(9 \times \left(x + \frac{4}{9}\right)\right)^2 \end{aligned}$$

$$\boxed{Q(x) = 81 \times \left(x + \frac{4}{9}\right)^2}$$

$$\begin{aligned} R(x) &= x^2 - 5x - 8 \\ &= \left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 8 \\ &= \left(x - \frac{5}{2}\right)^2 + \frac{-25}{4} - \frac{8 \times 4}{1 \times 4} \\ &= \left(x - \frac{5}{2}\right)^2 + \frac{-25}{4} - \frac{32}{4} \end{aligned}$$

$$\boxed{R(x) = \left(x - \frac{5}{2}\right)^2 + \frac{-57}{4}}$$

$$\begin{aligned} S(x) &= -3x^2 + x + 1 \\ &= -3 \times \left(x^2 - \frac{1}{3}x - \frac{1}{3}\right) \\ &= -3 \times \left(\left(x - \frac{1}{6}\right)^2 - \left(\frac{1}{6}\right)^2 + \frac{-1}{3}\right) \\ &= -3 \times \left(\left(x - \frac{1}{6}\right)^2 + \frac{-1}{36} + \frac{-1 \times 12}{3 \times 12}\right) \\ &= -3 \times \left(\left(x - \frac{1}{6}\right)^2 + \frac{-1}{36} + \frac{-12}{36}\right) \\ &= -3 \times \left(\left(x - \frac{1}{6}\right)^2 + \frac{-13}{36}\right) \\ &= -3 \times \left(x - \frac{1}{6}\right)^2 + \frac{-13 \times 3 \times (-1)}{3 \times 12} \end{aligned}$$

$$\boxed{S(x) = -3 \times \left(x - \frac{1}{6}\right)^2 + \frac{13}{12}}$$

Corrigé de l'exercice 4

Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= x^2 - 3x + 8 \\ &= \left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 8 \\ &= \left(x - \frac{3}{2}\right)^2 + \frac{-9}{4} + \frac{8 \times 4}{1 \times 4} \\ &= \left(x - \frac{3}{2}\right)^2 + \frac{-9}{4} + \frac{32}{4} \end{aligned}$$

$$\boxed{P(x) = \left(x - \frac{3}{2}\right)^2 + \frac{23}{4}}$$

$$\begin{aligned} Q(x) &= x^2 + 16x - 2 \\ &= (x+8)^2 - 8^2 - 2 \\ &= (x+8)^2 - 64 - 2 \end{aligned}$$

$$\boxed{Q(x) = (x+8)^2 - 66}$$

$$\begin{aligned} R(x) &= 4x^2 - 8x + 4 \\ &= (2x-2)^2 \\ &= \left(2 \times \left(x - \frac{2}{2}\right)\right)^2 \end{aligned}$$

$$\boxed{R(x) = 4 \times (x-1)^2}$$

$$\begin{aligned} S(x) &= 2x^2 - 8x + 7 \\ &= 2 \times \left(x^2 - 4x + \frac{7}{2}\right) \\ &= 2 \times \left((x-2)^2 - 2^2 + \frac{7}{2}\right) \\ &= 2 \times \left((x-2)^2 - 4 + \frac{7}{2}\right) \\ &= 2 \times \left((x-2)^2 + \frac{-4 \times 2}{1 \times 2} + \frac{7}{2}\right) \\ &= 2 \times \left((x-2)^2 + \frac{-8}{2} + \frac{7}{2}\right) \\ &= 2 \times \left((x-2)^2 + \frac{-1}{2}\right) \\ &= 2 \times (x-2)^2 + \frac{-1 \times 2}{2 \times 1} \end{aligned}$$

$$\boxed{S(x) = 2 \times (x-2)^2 - 1}$$

Corrigé de l'exercice 5

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = -2x^2 - x - 6$$

$$= -2 \times \left(x^2 + \frac{1}{2}x + 3 \right)$$

$$= -2 \times \left(\left(x + \frac{1}{4} \right)^2 - \left(\frac{1}{4} \right)^2 + 3 \right)$$

$$= -2 \times \left(\left(x + \frac{1}{4} \right)^2 + \frac{-1}{16} + \frac{3 \times 16}{1 \times 16} \right)$$

$$= -2 \times \left(\left(x + \frac{1}{4} \right)^2 + \frac{-1}{16} + \frac{48}{16} \right)$$

$$= -2 \times \left(\left(x + \frac{1}{4} \right)^2 + \frac{47}{16} \right)$$

$$= -2 \times \left(x + \frac{1}{4} \right)^2 + \frac{47 \times 2 \times (-1)}{2 \times 8}$$

$$P(x) = -2 \times \left(x + \frac{1}{4} \right)^2 + \frac{-47}{8}$$

$$Q(x) = 36x^2 - 60x + 25$$

$$= (6x - 5)^2$$

$$= \left(6 \times \left(x - \frac{5}{6} \right) \right)^2$$

$$Q(x) = 36 \times \left(x - \frac{5}{6} \right)^2$$

$$R(x) = x^2 + 14x - 2$$

$$= (x + 7)^2 - 7^2 - 2$$

$$= (x + 7)^2 - 49 - 2$$

$$R(x) = (x + 7)^2 - 51$$

$$S(x) = x^2 + 3x - 1$$

$$= \left(x + \frac{3}{2} \right)^2 - \left(\frac{3}{2} \right)^2 - 1$$

$$= \left(x + \frac{3}{2} \right)^2 + \frac{-9}{4} - \frac{1 \times 4}{1 \times 4}$$

$$= \left(x + \frac{3}{2} \right)^2 + \frac{-9}{4} - \frac{4}{4}$$

$$S(x) = \left(x + \frac{3}{2} \right)^2 + \frac{-13}{4}$$

Corrigé de l'exercice 6

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 2x^2 + 3x - 9$$

$$= 2 \times \left(x^2 + \frac{3}{2}x - \frac{9}{2} \right)$$

$$= 2 \times \left(\left(x + \frac{3}{4} \right)^2 - \left(\frac{3}{4} \right)^2 + \frac{-9}{2} \right)$$

$$= 2 \times \left(\left(x + \frac{3}{4} \right)^2 + \frac{-9}{16} + \frac{-9 \times 8}{2 \times 8} \right)$$

$$= 2 \times \left(\left(x + \frac{3}{4} \right)^2 + \frac{-9}{16} + \frac{-72}{16} \right)$$

$$= 2 \times \left(\left(x + \frac{3}{4} \right)^2 + \frac{-81}{16} \right)$$

$$= 2 \times \left(x + \frac{3}{4} \right)^2 + \frac{-81 \times 2}{2 \times 8}$$

$$P(x) = 2 \times \left(x + \frac{3}{4} \right)^2 + \frac{-81}{8}$$

$$Q(x) = 25x^2 - 70x + 49$$

$$= (5x - 7)^2$$

$$= \left(5 \times \left(x - \frac{7}{5} \right) \right)^2$$

$$Q(x) = 25 \times \left(x - \frac{7}{5} \right)^2$$

$$R(x) = x^2 - 18x + 3$$

$$= (x - 9)^2 - 9^2 + 3$$

$$= (x - 9)^2 - 81 + 3$$

$$R(x) = (x - 9)^2 - 78$$

$$S(x) = x^2 + 3x + 8$$

$$= \left(x + \frac{3}{2} \right)^2 - \left(\frac{3}{2} \right)^2 + 8$$

$$= \left(x + \frac{3}{2} \right)^2 + \frac{-9}{4} + \frac{8 \times 4}{1 \times 4}$$

$$= \left(x + \frac{3}{2} \right)^2 + \frac{-9}{4} + \frac{32}{4}$$

$$S(x) = \left(x + \frac{3}{2} \right)^2 + \frac{23}{4}$$

Corrigé de l'exercice 7

Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= x^2 - 2x - 5 \\ &= (x - 1)^2 - 1^2 - 5 \\ &= (x - 1)^2 - 1 - 5 \end{aligned}$$

$$\boxed{P(x) = (x - 1)^2 - 6}$$

$$\begin{aligned} Q(x) &= 49x^2 + 42x + 9 \\ &= (7x + 3)^2 \\ &= \left(7 \times \left(x + \frac{3}{7}\right)\right)^2 \end{aligned}$$

$$\boxed{Q(x) = 49 \times \left(x + \frac{3}{7}\right)^2}$$

$$\begin{aligned} R(x) &= 2x^2 - 3x - 4 \\ &= 2 \times \left(x^2 - \frac{3}{2}x - 2\right) \\ &= 2 \times \left(\left(x - \frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 - 2\right) \\ &= 2 \times \left(\left(x - \frac{3}{4}\right)^2 + \frac{-9}{16} - \frac{2 \times 16}{1 \times 16}\right) \end{aligned}$$

$$\begin{aligned} &= 2 \times \left(\left(x - \frac{3}{4}\right)^2 + \frac{-9}{16} - \frac{32}{16}\right) \\ &= 2 \times \left(\left(x - \frac{3}{4}\right)^2 + \frac{-41}{16}\right) \\ &= 2 \times \left(x - \frac{3}{4}\right)^2 + \frac{-41 \times 2}{2 \times 8} \end{aligned}$$

$$\boxed{R(x) = 2 \times \left(x - \frac{3}{4}\right)^2 + \frac{-41}{8}}$$

$$\begin{aligned} S(x) &= x^2 + 5x + 7 \\ &= \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 7 \\ &= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{7 \times 4}{1 \times 4} \\ &= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{28}{4} \end{aligned}$$

$$\boxed{S(x) = \left(x + \frac{5}{2}\right)^2 + \frac{3}{4}}$$

Corrigé de l'exercice 8

Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= x^2 - 18x - 6 \\ &= (x - 9)^2 - 9^2 - 6 \\ &= (x - 9)^2 - 81 - 6 \end{aligned}$$

$$\boxed{P(x) = (x - 9)^2 - 87}$$

$$\begin{aligned} R(x) &= 16x^2 + 64x + 64 \\ &= (4x + 8)^2 \\ &= \left(4 \times \left(x + \frac{8}{4}\right)\right)^2 \end{aligned}$$

$$\boxed{R(x) = 16 \times \left(x + 2\right)^2}$$

$$\begin{aligned} Q(x) &= -3x^2 - 8x + 5 \\ &= -3 \times \left(x^2 + \frac{8}{3}x - \frac{5}{3}\right) \\ &= -3 \times \left(\left(x + \frac{4}{3}\right)^2 - \left(\frac{4}{3}\right)^2 + \frac{-5}{3}\right) \\ &= -3 \times \left(\left(x + \frac{4}{3}\right)^2 + \frac{-16}{9} + \frac{-5 \times 3}{3 \times 3}\right) \end{aligned}$$

$$\begin{aligned} &= -3 \times \left(\left(x + \frac{4}{3}\right)^2 + \frac{-16}{9} + \frac{-15}{9}\right) \\ &= -3 \times \left(\left(x + \frac{4}{3}\right)^2 + \frac{-31}{9}\right) \\ &= -3 \times \left(x + \frac{4}{3}\right)^2 + \frac{-31 \times 3 \times (-1)}{3 \times 3} \end{aligned}$$

$$\boxed{Q(x) = -3 \times \left(x + \frac{4}{3}\right)^2 + \frac{31}{3}}$$

$$\begin{aligned} S(x) &= x^2 + 5x - 5 \\ &= \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 5 \\ &= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} - \frac{5 \times 4}{1 \times 4} \\ &= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} - \frac{20}{4} \end{aligned}$$

$$\boxed{S(x) = \left(x + \frac{5}{2}\right)^2 + \frac{-45}{4}}$$